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Thermoporoelastic response of a fluid-saturated porous sphere: An analytical solution

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ABSTRACT

The fully coupled thermo/hydro/mechanical response of a fluid saturated porous sphere subject to a pressure stress pulse on the outer boundary is considered. A full analytical method is developed and an exact unique solution of the coupled equations is presented. This generality allows us to simulate a variety of practical problems.

1. Introduction

We are interested in an adaptive material that can change its properties autonomously in response to an external generated signal. It can be a mechanical, electrical or thermal signal. Mechanical stress causes temperature and fluid pressure changes in the media. To this end, we study temperature and fluid pressure distributions, and their time evolution, caused by a mechanical pulse.

Very few analytical solutions of the fully coupled thermoporoelastic equations are currently available. Typically, the solutions are derived under the assumptions that some of the couplings can be neglected (see e.g., McTigue (1986), Coussy (2004)). For instance, in Coussy (2004), an analytical solution is presented for a half-space subjected to a change in temperature, under the assumption that the temperature equation can be decoupled and approximated by a purely diffusive equation. Kodashima and Kurashige (1996, 1997) analyzed thermal stresses induced in a fluid-saturated porous hollow elastic sphere subjected to a sudden rise in temperature and pressure on its inner wall. However in their work the displacement field is decoupled from the temperature and pore fluid pressure fields. Furthermore, to avoid analytical solution of equations with nonlinear and integral terms, the authors use a numerical method to solve the system. To understand and investigate all thermo/hydro/mechanical coupling effects, we are interested in fully analytical solution of a porous sphere under a radial stress pulse on the outer boundary. The goal is to compute temperature and fluid pressure distributions in the porous sphere, their evolution with time, in order to understand the mechanisms of heat and fluid transports and to study the influence of the various physical parameters on the solution.

Cryer (1963) previously solved this problem assuming isothermal conditions, an incompressible fluid, and incompressible solid grains (viz., Biot b coefficient, $b = 1$).

In this work an analytical method is employed to obtain the response of a fully saturated porous solid sphere under mechanical pulse load. The method of solution is based on the Laplace transformation method. The solution is deduced using the Laplace inversion formulas with the theorem of residue.

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In the following we first present the mathematical formulation of the problem based on Coussy (2004) theory for a porous medium. In the next section the basic equations of the theory are described and specialized to the problem in hand. Afterwards analytical expressions for the fluid pressure and temperature at the center are derived, and some numerical results are presented.

2. Mathematical formulation of a problem

A general treatment of the thermomechanical behavior of a porous solid is presented by Coussy (2004) and is an extension of the classical work by Chadwick (1960), Eringen (1980) and Novacki (1966) in thermoelasticity of solid. The basic equations for a porous elastic medium are the stress balance of momentum equation:

$$(\lambda + 2\mu)\nabla(\nabla \cdot \underset{\sim}{u}) - \mu\nabla \times (\nabla \times \underset{\sim}{u}) - \beta\nabla T - b\nabla p + \rho \underset{\sim}{g} = 0 \tag{1}$$

together with the energy balance equation:

$$-\kappa\nabla^2 T - 3\alpha_m T_0 \dot{p} + T_0\beta\nabla \cdot \underset{\sim}{\dot{u}} + \rho\gamma\dot{T} = 0 \tag{2}$$

and the fluid mass balance equation:

$$-k\nabla^2 p - \frac{1}{M}\dot{p} + b\nabla \cdot \underset{\sim}{\dot{u}} - 3\alpha_m \dot{T} = 0 \tag{3}$$

where a superimposed dot is used to indicate a time derivative, and the following notation is used:

g	acceleration of gravity
λ, μ	Lame coefficients of the (drained) porous skeleton
p	fluid pressure
T	temperature
b	Biot's coefficient (Biot (1955)); $b = 1 - \frac{K}{K_S}$ $K = \lambda + \frac{2\mu}{3}$ =bulk modulus
K_S	bulk modulus of the solid grains
$\underset{\sim}{u}$	displacement vector of the skeleton
$e = \nabla \cdot \underset{\sim}{u}$	dilatation of the skeleton
β	$3\alpha K$
α	coefficient of linear solid thermal expansion
α_f	coefficient of linear fluid thermal expansion
α_m	$(b - \varphi)\alpha + \varphi\alpha_f$
φ	*porosity*
κ	thermal conductivity of the porous skeleton
k_s	intrinsic permeability
$k = k_S/\mu_f$	*mobility*
μ_f	fluid viscosity
ρ	$(1 - \varphi)\rho_s + \varphi\rho_f$
ρ_s, ρ_f	skeleton and fluid mass densities
γ_s, γ_f	skeleton and fluid specific heats
$\rho\gamma$	$(1 - \varphi)\rho_s\gamma_s + \varphi\rho_f\gamma_f$
$\frac{1}{M}$	$\frac{b-\varphi}{K_s} + \frac{\varphi}{K_f}$
K_f	fluid bulk modulus

Introducing spherical coordinates r, θ, φ, the radial displacement is denoted by u_r. Because of spherical symmetry, it follows from the identity,

$$e = \nabla \cdot \underset{\sim}{u} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r) \quad \text{and} \quad \nabla(\nabla \cdot u) = \frac{\partial}{\partial r}(\nabla \cdot u) \tag{4}$$

The equations for a porous medium sphere (1,2,3) reduce to:

$$(\lambda + 2\mu)\nabla e - \beta\nabla T - b\nabla p = 0 \tag{5}$$

$$-\kappa\nabla^2 T + T_0\beta\dot{e} + \rho\gamma\dot{T} - 3\alpha_m T_0\dot{p} = 0 \tag{6}$$

$$-k\nabla^2 p + b\dot{e} - 3\alpha_m\dot{T} - \frac{1}{M}\dot{p} = 0 \tag{7}$$

Instantaneous mean stress response

The instantaneous response for the dilatation, temperature and fluid pressure in the sphere center as a result of a radial stress jump $\Delta\sigma_r = -P$ on the outer boundary, can be deduced from Eqs. (5)–(7) as follows:

$$\Delta e = \frac{\Delta\sigma(\rho\gamma - 9M\alpha_m^2 T_0)}{D} \tag{8}$$

$$\Delta T = \frac{-\Delta\sigma 3T_0(bM\alpha_m + \alpha K)}{D} \tag{9}$$

$$\Delta p = \frac{-\Delta\sigma M(9\alpha\alpha_m K T_0 + b\rho\gamma)}{D} \tag{10}$$

$$D = 6\alpha_m\beta T_0 M b + \rho\gamma b^2 M + T_0\beta^2 - 9\alpha_m^2 T_0 M K + \rho\gamma K \tag{11}$$

Now we are interested in the pressure and temperature time evolution at the center of the porous sphere of radius R under the following conditions:

(1) fluid drainage on the outer boundary (viz, $p(R, t = 0^+) = 0$)
(2) temperature remains constant on the outer boundary (viz, $T(R, t = 0^+) = T_0$)

Presentation of equations

Introducing function υ and u,

$$\upsilon = \frac{T_0\beta}{\kappa} e + \frac{\rho\gamma}{\kappa} T - \frac{3\alpha_m T_0}{\kappa} p \tag{12}$$

$$u = \frac{b}{k} e - \frac{3\alpha_m}{k} T + \frac{1}{Mk} p \tag{13}$$

and expressing e from (5)

$$\nabla^2 e = \frac{b}{\lambda + 2\mu}\nabla^2 p + \frac{\beta}{\lambda + 2\mu}\nabla^2 T \tag{14}$$

we obtain next equations

$$\nabla^2\upsilon = \frac{dT_0}{\kappa}\nabla^2 p + \frac{1}{C_\theta}\nabla^2 T \tag{15}$$

$$\nabla^2 u = \frac{1}{C_f}\nabla^2 p + \frac{d}{\kappa}\nabla^2 T \tag{16}$$

where we note fluid diffusion coefficient C_f, thermal diffusion coefficient C_θ, and d

$$C_f = kM\frac{\lambda + 2\mu}{(\lambda + 2\mu) + b^2 M} \tag{17}$$

$$C_\theta = \frac{\kappa}{\rho\gamma}\frac{\lambda + 2\mu}{(\lambda + 2\mu) + \frac{T_0\beta^2}{\rho\gamma}} \tag{18}$$

$$d = \frac{b\beta}{\lambda + 2\mu} - 3\alpha_m \tag{19}$$

We can express Eqs. (11) and (12) in the terms of υ and u

$$\begin{cases} \dot{\upsilon} = \nabla^2 T \\ \dot{u} = \nabla^2 p \end{cases} \Rightarrow \begin{cases} \nabla^2\upsilon = \frac{1}{C_\theta}\dot{\upsilon} + \frac{dT_0}{\kappa}\dot{u} \\ \nabla^2 u = \frac{d}{k}\dot{\upsilon} + \frac{1}{C_f}\dot{u} \end{cases} \tag{20}$$

We can rewrite this system of diffusion equation in a matrix form

$$\begin{pmatrix} \nabla^2\upsilon \\ \nabla^2 u \end{pmatrix} = \begin{pmatrix} \frac{1}{C_\theta} & \frac{dT_0}{\kappa} \\ \frac{d}{k} & \frac{1}{C_f} \end{pmatrix}\begin{pmatrix} \dot{\upsilon} \\ \dot{u} \end{pmatrix} \Rightarrow \nabla^2\mathbf{V} = \mathbf{J}^{-1}\dot{\mathbf{V}} \Rightarrow \dot{\mathbf{V}} = \mathbf{J}\nabla^2\mathbf{V} \tag{21}$$

Change of variables

$$\upsilon' = \upsilon - \frac{\rho\gamma T_0}{\kappa} \quad \text{and } u\prime = u + \frac{3\alpha_m T_0}{k}$$

give a new matrix equation

$$\dot{\mathbf{V}}' = \mathbf{J}\nabla^2\mathbf{V}' \tag{22}$$

The initial and boundary conditions are that the strains are zero and the temperature is T_0 initially, the temperature is T_0 at surface and that the normal stress at the surface is – P. These conditions can be written

Initial conditions:

$$\begin{aligned} &T = T_0 \\ &p = 0 \quad \text{for } t = 0 \\ &e = 0 \end{aligned} \tag{23}$$

Boundary conditions:

$$\begin{aligned} &T = T_0 \\ &p = 0 \quad \text{for } r = R, t > 0 \\ &\sigma_r = -P \end{aligned} \tag{24}$$

where

$$\begin{aligned} &\sigma_r = (\lambda + 2\mu)e - \frac{4\mu}{r^3}\int_0^r er^2 dr \Rightarrow -P = (\lambda + 2\mu)e(R,t) - \frac{4\mu}{R^3}\int_0^R er^2 dr \\ &e(R,t) = \frac{4\mu}{(\lambda + 2\mu)R^3}\int_0^R er^2 dr - \frac{P}{\lambda + 2\mu} \end{aligned} \tag{25}$$

To obtain the boundary conditions in terms of υ and u, we integrate Eq. (5) and substituting in the Eqs. (12) and (13) we express υ and u in terms of e and T

$$\begin{aligned} &\upsilon' = \left[\frac{T_0\beta}{\kappa} - \frac{3\alpha_m T_0(\lambda + 2\mu)}{\kappa b}\right]e + \left[\frac{\rho\gamma}{\kappa} + \frac{3\alpha_m T_0\beta}{\kappa b}\right]T + \frac{3\alpha_m T_0(\lambda + 2\mu)}{\kappa b}e(R,t) - \left[\frac{\rho\gamma}{\kappa} + \frac{3\alpha_m T_0\beta}{\kappa b}\right]T_0 \\ &u' = \left[\frac{b}{k} + \frac{\lambda + 2\mu}{bMk}\right]e - \left[\frac{3\alpha_m}{k} + \frac{\beta}{bMk}\right]T - \frac{\lambda + 2\mu}{bMk}e(R,t) + \left[\frac{3\alpha_m}{k} + \frac{\beta}{bMk}\right]T_0 \end{aligned} \tag{26}$$

We can rewrite system of Eq. (26) in the matrix form

$$\begin{pmatrix} \upsilon' \\ u' \end{pmatrix} = \begin{pmatrix} A_1 & A_2 \\ B_1 & -B_2 \end{pmatrix}\begin{pmatrix} e \\ T \end{pmatrix} + \begin{pmatrix} A_3 & -A_2 \\ -B_3 & B_2 \end{pmatrix}\begin{pmatrix} e(R,t) \\ T(R,t) \end{pmatrix} \Rightarrow \mathbf{V}' = \mathbf{AE} + \mathbf{BE}(R,t) \tag{27}$$

$$\mathbf{E} = \mathbf{A}^{-1}\mathbf{V}' - \mathbf{A}^{-1}\mathbf{BE}(R,t) \tag{28}$$

Boundary conditions (24) and (25) give

$$\mathbf{E}(R,t) = \begin{pmatrix} \frac{4\mu}{(\lambda+2\mu)R^3} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \int_0^R er^2 dr \\ \int_0^R Tr^2 dr \end{pmatrix} + \begin{pmatrix} -\frac{P}{\lambda+2\mu} \\ T_0 \end{pmatrix} = \mathbf{D}\int_0^R \mathbf{E}r^2 dr + \mathbf{F} \tag{29}$$

Substituting (28) into (29) we obtain

$$\begin{aligned} &\mathbf{E}(R,t) = \left(\mathbf{I} + \mathbf{DA}^{-1}\mathbf{B}\frac{R^3}{3}\right)^{-1}\mathbf{DA}^{-1}\int_0^R \mathbf{V}'r^2 dr + \left(\mathbf{I} + \mathbf{DA}^{-1}\mathbf{B}\frac{R^3}{3}\right)^{-1}\mathbf{F} \\ &\mathbf{V}'(R,t) = (\mathbf{A} + \mathbf{B})\left(\mathbf{I} + \mathbf{DA}^{-1}\mathbf{B}\frac{R^3}{3}\right)^{-1}\mathbf{DA}^{-1}\int_0^R \mathbf{V}'r^2 dr + (\mathbf{A} + \mathbf{B})\left(\mathbf{I} + \mathbf{DA}^{-1}\mathbf{B}\frac{R^3}{3}\right)^{-1}\mathbf{F} = \mathbf{H}\int_0^R \mathbf{V}'r^2 dr + \mathbf{F}' \end{aligned} \tag{30}$$

Now we can write initial and boundary conditions for the matrix diffusion Eq. (22):

$$\mathbf{V}'(r,0) = 0 \tag{31}$$

$$\mathbf{V}'(R,t) = \mathbf{H}\int_0^R \mathbf{V}'r^2 dr - \mathbf{F}' \tag{32}$$

3. Solution of equations

We solve Eq. (22) with initial and boundary conditions (31) and (32) with the aid of Laplace transforms.

It convenient to introduce an auxiliary variable (Cryer, 1963)

$$\mathbf{X}(r,t) = r\mathbf{V}'(r,t) \tag{33}$$

in terms of which Eq. (22) with their initial and boundary conditions become:

$$\dot{\mathbf{X}} = \mathbf{J}\nabla^2\mathbf{X} \tag{34}$$

$$\mathbf{X}(r,0) = 0 \tag{35}$$

$$\mathbf{X}(R,t) = R\mathbf{H}\int_0^R \mathbf{X} r dr - R\mathbf{F}' \tag{36}$$

$$\mathbf{X}(0,t) = 0 \tag{37}$$

Table 1
Reference parameters for the rock.

Reference parameters:
φ = 0.2
b=1
v = 0.25
E=10000 MPa
$\rho\gamma$ = 2.6 MJ/(m^3 $\circ$ C)
α = 1.2 × 10^{-5} °C^{-1}
α_f = 5 × 10^{-5} °C^{-1}
κ = 2.5 × 10^{-6} MW/m °C
K_f = 2170 MPa
k = 10^{-13} m^2
μ_f = 10^{-9} MPa s
C_f = 0.57 m^2/s
C_0 = 9.6 10^{-7} m^2/s

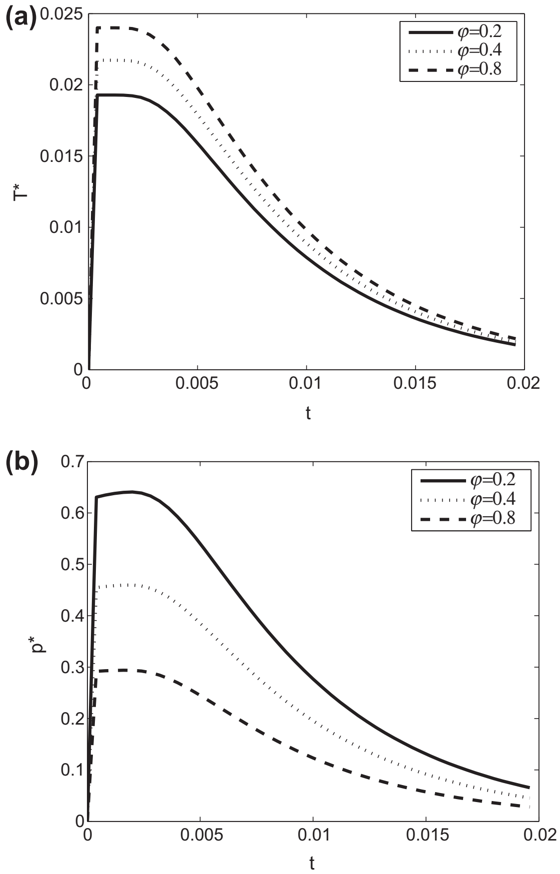

Fig. 1. Temperature (a) and pressure (b) distributions for different porosity φ = 0.2, 0.4, 0.8.

The Laplace transform will be denoted by a bar and the variable s instead of t (Carslaw & Jaeger, 1959), thus:

$$\widetilde{\mathbf{X}}(r,s)=\int_0^\infty \mathbf{X}(r,t)\exp(-st)dt \tag{38}$$

Transforming Eq. (34), and using condition (35), we obtain an ordinary differential equation for $\widetilde{\mathbf{X}}$

$$s\widetilde{\mathbf{X}}=\mathbf{J}\nabla^2\widetilde{\mathbf{X}} \;\Rightarrow\; \nabla^2\widetilde{\mathbf{X}}=\mathbf{J}^{-1}s\widetilde{\mathbf{X}} \tag{39}$$

with conditions

$$\widetilde{\mathbf{X}}(R,s)=R\mathbf{H}\int_0^R \widetilde{\mathbf{X}}(r,s)rdr-\frac{R\mathbf{F}'}{s} \tag{40}$$

$$\widetilde{\mathbf{X}}(0,s)=0 \tag{41}$$

Solving Eqs. (39)–(41), and using Eq. (33) we obtain the solution for $\widetilde{\mathbf{V}}'$

$$\widetilde{\mathbf{V}}'(r,s)=-\frac{\sinh(\mathbf{Q}r)}{r}\left[\mathbf{H}'\sinh(\mathbf{Q}R)+\mathbf{Q}^2\sinh(\mathbf{Q}R)-\mathbf{H}'\mathbf{Q}R\cosh(\mathbf{Q}R)\right]^{-1}\mathbf{J}^{-1}\mathbf{F}'R \tag{42}$$

where $\mathbf{Q}\equiv\ (\mathbf{J}^{-1}s)^{-1}$

Then, taking into account that

$$\mathbf{E}=\mathbf{A}^{-1}\mathbf{V}'-\mathbf{A}^{-1}\mathbf{V}'(R,t)+\mathbf{E}(R,t) \;\Rightarrow\; \widetilde{\mathbf{E}}(r,s)=\mathbf{A}^{-1}\left[\widetilde{\mathbf{V}}'(r,s)-\widetilde{\mathbf{V}}'(R,s)\right]+\frac{\mathbf{E}(R,t)}{s} \tag{43}$$

we obtain

$$\widetilde{\mathbf{E}}(0,s)=\mathbf{A}^{-1}[-\mathbf{Q}R+\sinh(\mathbf{Q}R)]\left[(\mathbf{H}'+\mathbf{Q}^2)\sinh(\mathbf{Q}R)-\mathbf{H}'\mathbf{Q}R\cosh(\mathbf{Q}R)\right]^{-1}\frac{\mathbf{J}^{-1}\mathbf{F}'R}{R}+\frac{\mathbf{E}(R,t)}{s} \tag{44}$$

where

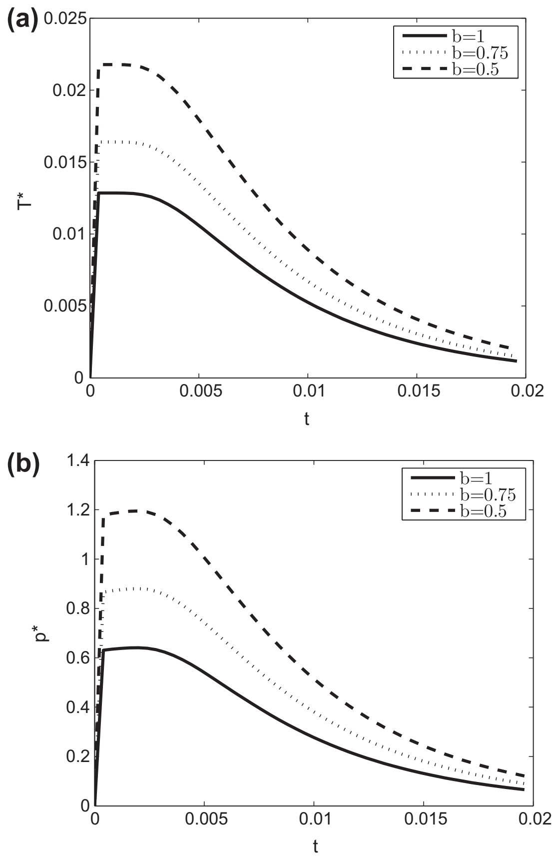

Fig. 2. Temperature (a) and pressure (b) distributions for b = 0.5, 0.75, 1.

$$\mathbf{H}' = \frac{4\mu}{D(\lambda+2\mu)R^2}\begin{pmatrix} T_0\beta(3\alpha_m bM+\beta) & \frac{T_0\beta(\rho\gamma b+3\alpha_m bT_0\beta)Mk}{\kappa} \\ \frac{b(3\alpha_m bM+\beta)\kappa}{k} & b(\rho\gamma b+3\alpha_m bT_0\beta)M \end{pmatrix} \tag{45}$$

It only remains to invert $\widetilde{\mathbf{E}}(0,s)$ using the standard technique of expansion in terms of residues (Carslaw & Jaeger, 1959). All relevant conditions are satisfied. Here the poles of integral are the values s_n,

$$\boldsymbol{\alpha}_n^2 = -J^{-1}s_n$$

where $\pm\alpha_n$, $(n = 1,2,3,\ldots)$ are the roots of $(\mathbf{H}' + \mathbf{Q}^2)\sinh(\mathbf{Q}R) - \mathbf{H}'\mathbf{Q}R\cosh(\mathbf{Q}R) = 0$

To find roots MATLAB solution was used.

$$\mathbf{E}(0,t) = 2\mathbf{A}^{-1}\sum_{n=1}^{\infty}\frac{(\mathbf{H}'-\boldsymbol{\alpha}_n^2)\frac{1}{\cos(\boldsymbol{\alpha}_n R)}-\mathbf{H}'}{(\boldsymbol{\alpha}_n^2-3\mathbf{H}'+R^2\mathbf{H}'^2)}e^{-\mathbf{J}\alpha_n^2 t}\mathbf{F}' + \mathbf{E}(R,t) \tag{46}$$

where

$$\mathbf{A}^{-1}\mathbf{F}' = -\begin{pmatrix} \frac{6\alpha_m MbT_0\beta+T_0\beta^2+\rho\gamma Mb^2}{(\lambda+2\mu)D}P \\ \frac{\beta+3\alpha_m Mb}{D}PT_0 \end{pmatrix} \tag{47}$$

$$D = 6\alpha_m\beta T_0 Mb + \rho\gamma b^2 M + T_0\beta^2 - 9\alpha_m^2 T_0 MK + \rho\gamma K \tag{48}$$

In the vector (47) the second component responds for the temperature that is consistent with expression for instantaneous response for ΔT in (9).

Pressure response

In the same way as in Eqs. (26)–(28) we obtain the equations that express υ and u in terms of e and p:

$$\begin{pmatrix} \upsilon' \\ u' \end{pmatrix} = \begin{pmatrix} A_1 & A_2 \\ B_1 & -B_2 \end{pmatrix}\begin{pmatrix} e \\ p \end{pmatrix} + \begin{pmatrix} A_3 & -A_2 \\ -B_3 & B_2 \end{pmatrix}\begin{pmatrix} e(R,t) \\ p(R,t) \end{pmatrix} \Rightarrow \mathbf{V}' = \mathbf{A}_p\mathbf{E}_p + \mathbf{B}\mathbf{E}_p(R,t) \tag{49}$$

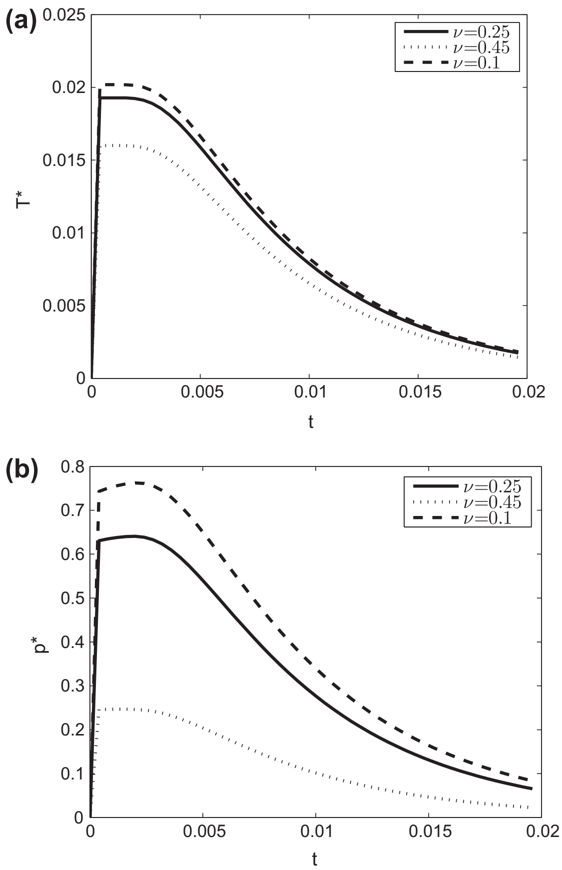

Fig. 3. Temperature (a) and pressure (b) distributions for ν = 0.1, 0.25, 0.45.

with

$$A_1 = \left[\frac{T_0\beta}{\kappa} + \frac{\rho\gamma(\lambda+2\mu)}{\kappa\beta}\right] \quad A_2 = -\left[\frac{3\alpha_m T_0}{\kappa} + \frac{\rho\gamma b}{\kappa\beta}\right] \quad A_3 = -\frac{\rho\gamma(\lambda+2\mu)}{\kappa\beta}$$
$$B_1 = \left[\frac{b}{k} - \frac{3\alpha_m(\lambda+2\mu)}{k\beta}\right] \quad B_2 = -\left[\frac{1}{Mk} + \frac{3\alpha_m b}{k\beta}\right] \quad B_3 = -\frac{3\alpha_m(\lambda+2\mu)}{\beta k} \tag{50}$$
$$\mathbf{E}_p = \mathbf{A}_p^{-1}\mathbf{V}' - \mathbf{A}_p^{-1}\mathbf{B}_p\mathbf{E}_p(R,t) \tag{51}$$

After the same operations as above for temperature computations we obtain an expression in sphere center for new vector

$$\mathbf{E}_p(0,t) = 2\mathbf{A}_p^{-1}\sum_{n=1}^{\infty}\frac{\left(\mathbf{H}'_p - \boldsymbol{\alpha}_n^2\right)\frac{1}{\cos(\boldsymbol{\alpha}_n R)} - \mathbf{H}'_p}{\left(\boldsymbol{\alpha}_n^2 - 3\mathbf{H}'_p + R^2\mathbf{H}'^2_p\right)} e^{-\mathbf{J}\alpha_n^2 t}\mathbf{F}'_p + \mathbf{E}_p(R,t) \tag{52}$$

where

$$\mathbf{A}_p^{-1}\mathbf{F}'_p = -\begin{pmatrix} \frac{6\alpha_m MbT_0\beta + T_0\beta^2 + \rho\gamma Mb^2}{(\lambda+2\mu)D} P \\ \frac{b\rho\gamma + 3\alpha_m\beta T_0}{D} MP \end{pmatrix} \tag{53}$$

In the vector (53) the second component responds for the temperature that is consistent with expression for instantaneous response for Δp in (10).

4. Results

The analytical solution has been validated by comparison with numerical finite element solution of the problem (Prevost, 1981). As an example we consider as a reference a porous sphere of rock material saturated with water with radius 4 m. The

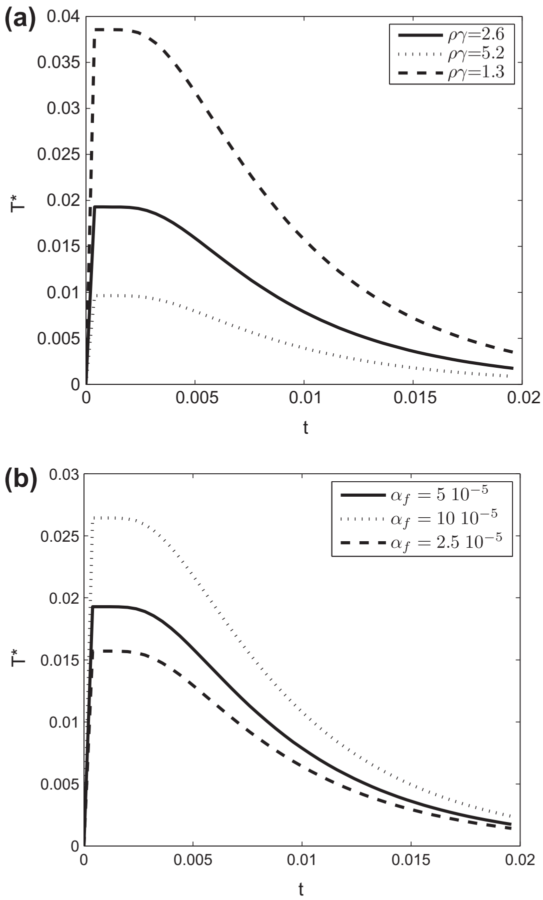

Fig. 4. Temperature distributions for $\rho\gamma = 1.3, 2.6, 5.2$ (a) and for $\alpha_f = 2.5\times10^{-5}, 5\times10^{-5}, 10\times10^{-5}$.

parameters are given in the Table 1. Initial temperature is considered to be 20 °C and applied pressure 1000 MPa. The reference case is presented with a solid line in Figs. 1–4. We then vary some of the parameters such as porosity φ, Biot coefficient b, Poison ratio ν, specific heat γ and coefficient of linear fluid thermal expansion α_f to investigate their influence on the solution.

We present the solutions for non-dimensional temperature $T^* = (T(0,t) - T_0)/T_0$ and pressure $p^* = p(0,t)/P$ at the scale of the corresponding non-dimensional time $C_\theta t/R^2$ for temperature, and $C_f t/R^2$ for pressure, respectively.

Fig. 1 shows the effect of porosity φ on the pressure evolution. Pressure decreases with increase in porosity. Opposite tendency is observed for the temperature and its effect is attenuated.

Figs. 2 and 3 show the growth of the temperature and pressure values. These values increase with the increase of Poisson's ratio ν and Biot coefficient b.

Fig. 3 shows that Poison ratio ν is an important parameter which has influence on the solution. It concerns both temperature and pressure increases, their shapes and their time evolution. For high Poison ratio the pressure tends to increase for some time while for the low expansion it immediately decreases. The pressure increase is referred to as the Mandel–Cryer effect (Mandel, 1953; Cryer, 1963), as observed in the isothermal case. Influence on the temperature is more attenuated.

Fig. 4 shows influence of parameters such as specific heat γ and coefficient of linear fluid thermal expansion α_f on the temperature. Influence of these parameters on the pressure is negligible.

5. Conclusion

An analytical solution for the coupled thermoporoelastic response of a fluid saturated porous sphere under stress pulse on its outer boundary is presented. We use the Laplace transform and Laplace inversion formulas with the theorem of residue. Advantage of the analytical solution is its ability to reveal the fundamental mathematical and physical features of the solution and allows interpretations of the parameters effects.

Temperature and pressure time evolutions and their spatial distributions can be controlled by varying the system parameters such as thermal expansion, specific heat, etc.

The analytical solution was validated by comparing it to finite elements solution (Prevost, 1981) of the problem. The MATLAB program is available at http://www.princeton.edu/~dynaflow/pub/.

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